

## Lec 5

Find distance between two vertices.

①. DFS fails.

②. BFS. Breadth-First Search.

BFS( $G, s$ ).

for all  $u \in V$  :  $\text{dist}(u) = \infty$ .

$\text{dist}(s) = 0$ .

$Q = [s]$

while  $Q$  is not empty

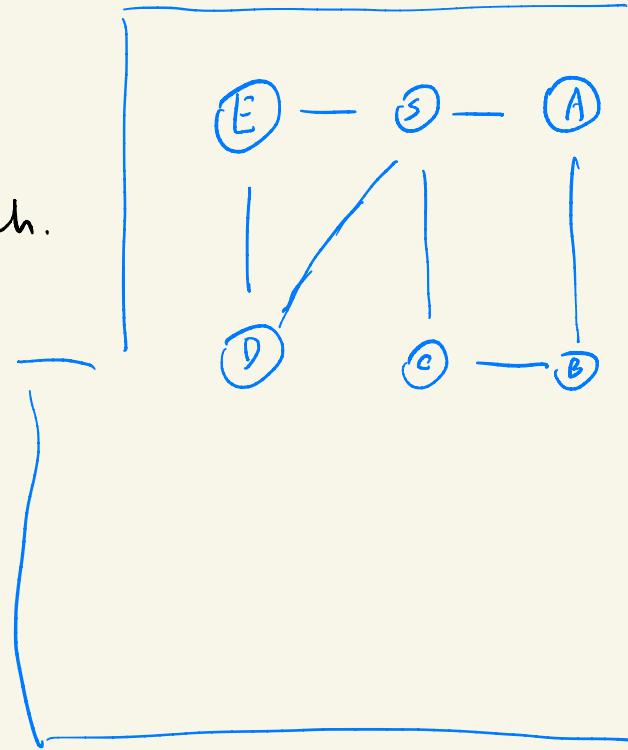
$u = \text{eject}(Q)$ .

for all  $(u, v) \in E$

if  $\text{dist}(v) = \infty$

$\text{inject}(Q, v)$ .

$\text{dist}(v) = \text{dist}(u) + 1$ .



## Correctness

Inductive proof.

- If d
- ① all nodes at distance  $\leq d$  correctly set
  - ② all others dist =  $\infty$
  - ③ at some time  $Q$  contains exactly nodes at dist d

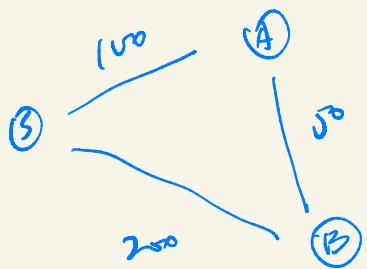
DFS v.s. BFS.

Stack                  Queue.

Time                   $O(|V| + |E|)$ .

## Edge with Length

BFS : Break edge into dummy vertices.



Set alarm for  $s$  at time 0.

Repeat until no alarm  
a) alarm at  $t$   
for  $\forall (u, v) \in E$   
update alarm of  $v$ .

## Dijkstra

for all  $u \in V$

$$\text{dist}(u) = \infty.$$

$$\text{prev}(u) = -.$$

$$\text{dist}(s) = 0.$$

$H = \text{makequeue}(V)$ .

while  $H$  is not empty.

$u = \text{deletemin}(H)$ .

for all  $(u, v) \in E$ .

if  $\text{dist}(v) > \text{dist}(u) + l(u, v)$

$$\text{dist}(v) = \text{dist}(u) + l(u, v)$$

$$\text{prev}(v) = u.$$

$\text{decreasekey}(H, v)$ .

Dijkstra = BFS with priority Queue !.

Another view of Dijkstra's algo.

"Grow the known region."

Dijkstra.

$\text{dist}(\cdot) = \infty$ ,  $\text{dist}(s) = 0$ .

while  $R \neq V$ .

Pick  $v \notin R$  with smallest  $\text{dist}(\cdot)$

Add  $v$  to  $R$

for all  $(v, w) \in E$ .

if  $\text{dist}(w) > \text{dist}(v) + l(v, w)$

$\text{dist}(w) = \text{dist}(v) + l(v, w)$ .

Correctness by induction (Exercise).

Running time.  $|V| \text{ delete min} + |V| + |E| / \text{modify}$

## Implementation

	delete min	modify insert	total
Array	$O( V )$	$O(1)$	$O( V ^2)$
Binary Heaps	$O(\log  V )$	$O(\log  V )$	$O(( V  +  E ) \log  V )$
d-ary Heap	$\frac{d \log  V }{\log d}$	$O\left(\frac{\log  V }{\log d}\right)$	$O\left(( V  \cdot d +  E ) \frac{\log  V }{\log d}\right)$
Fibonacci Heap	$O(\log  V )$	$O\left( E  \cdot \frac{\log  V }{\log  E  - \log  V }\right)$	$d = \frac{ E }{ V } \Rightarrow$ $ E  = O( V ) \Rightarrow O( V  \log  V )$ $ E  = \Theta( V ^2) \Rightarrow O( V ^2)$ $ E  =  V ^{1+\delta} \downarrow O( E )$

## Negative Weight

update  $(u, v)$

$$\text{dist}(v) = \min \{ \text{dist}(v), \text{dist}(u) + \ell(u, v) \}.$$

\*). update is harmless.

\*). give correct answer if updating in correct order



Bellman-Ford: Update all edges  $|V|-1$  times!

## Shortest path in DAG

① Topo sort

② Update in the order